"Holograms in the extreme edge illumination geometry"

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ABSTRACT

The recording of holograms in which the reference beam enters through the supporting substrate involves attenuation factors which differ strongly between signal and reference. This paper addresses the issue of fringe contrast under these extreme conditions and provides quantitative data that are indicative of at least a partial solution to the problem. We define a spatial average for the fringe contrast and offer results which follow our theoretical model for Du Pont OmniDex™ photopolymers. The direct edge referenced technique for recording is re-visited with observation of a new phenomenon which we call self-induced index matching. An alternative fringe recording geometry is proposed and compared with direct recording.

Keywords: Hologram; Edge-lit; Edge-Illuminated; Beam Ratio; Du Pont Photopolymers; Index Matching; HOE; Attenuation; Fluorescence; Swelling

1. INTRODUCTION

The newest technology often strives to make devices smaller and more compact. With holography's capability to combine large optical elements into one thin film, many optical systems can be dramatically reduced in size. Edge-illuminated holograms have maintained their appeal due to the potential for very compact illumination. The display hologram industry has continually dreamed of moving the light source for the hologram inside the actual frame, as is possible with an edge-illuminated hologram. Edge-illuminated Holographic Optical Elements (HOE's) have introduced many markets such as fingerprint identification[1] and helmet mounted displays to the possibilities of miniaturization.

2. GENERAL EDGE-ILLUMINATED HOLOGRAPHY

Edge-illuminated holography has been researched by many groups, such as Upatnieks[2], Benton[3], Caulfield[4], and Phillips[5], and others. While some of these groups have taken different approaches, we wish to consider a general case looking at the contrast with particular attention to the absorption problems when using photopolymers. With the assumption that fringe contrast is directly related to index modulation when exposing into the photopolymers, we can assume that the maximum average fringe contrast will give us the maximum diffraction efficiency. We shall first model the fringe contrast by looking at the superposition of a reference and signal wave in a general edge-referenced reflection hologram geometry as shown in Figure 1.

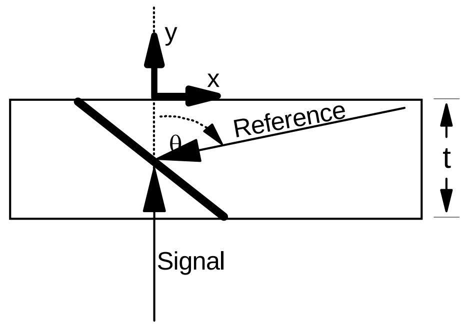

FIGURE 1. The diagram of a general case edge-referenced reflection hologram with the signal incident normal to the recording film. (Note: The substrate is not included for reasons of simplicity and the signal is perpendicular to the surface for our illustrations here).

Using the diagram in Figure 1 we can deduce the equations for the reference and signal waves.

$$A_s e^{ik_m y - \alpha(y+t)} \quad \text{and} \quad A_r e^{-ik_m x \sin\theta + \frac{\alpha y}{\cos\theta}} \tag{1}$$

These functions represent the signal and reference waves respectively with A_s and A_r as the signal and reference amplitudes, α the absorption coefficient, k_m the wave propagation constant in the medium, and θ the angle of the reference beam from the surface normal inside the film. The total amplitude at any point in the hologram is thus

$$A_{tot} = A_s e^{ik_m y - \alpha(y+t)} + A_r e^{-ik_m x \sin\theta + \frac{\alpha y}{\cos\theta}} \tag{2}$$

and the intensity is expressed as

$$I = A_{tot} \times A_{tot}^* = A_s^2 e^{-2\alpha(y+t)} + A_r^2 e^{\frac{2\alpha y}{\cos\theta}} + 2 A_s A_r e^{-\alpha(y+t) + \frac{\alpha y}{\cos\theta}} \times \cos(k_m y + k_m x \sin\theta) \tag{3}$$

As contrast is defined as

$$C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}},$$

we can deduce the fringe contrast for any point within the hologram, thus yielding (4)

$$C = \frac{2A_s A_r e^{-\alpha(y+t)+\frac{\alpha y}{\cos\theta}}}{A_s^2 e^{-2\alpha(y+t)} + A_r^2 e^{\frac{2\alpha y}{\cos\theta}}}. \tag{5}$$

This equation can be simplified by multiplying the numerator and denominator by $e^{\alpha y - \frac{\alpha y}{\cos\theta} + \alpha t}$ to yield the result

$$C = \frac{2A_s A_r}{A_s^2 e^{-\alpha y - \alpha t - \frac{\alpha y}{\cos\theta}} + A_r^2 e^{\alpha y + \alpha t + \frac{\alpha y}{\cos\theta}}}. \tag{6}$$

We now wish to find the average contrast $\langle C \rangle = \frac{1}{t}\int_{-t}^{0} C\, dy$.

By letting $z = e^{\alpha y + \alpha t + \frac{\alpha y}{\cos\theta}}$ and $\frac{\partial z}{\partial y} = \left(\alpha + \frac{\alpha}{\cos\theta}\right) z$ we can write

$$\langle C \rangle = \frac{1}{\alpha t\left(1 + \frac{1}{\cos\theta}\right)} \int_{e^{\frac{-\alpha t}{\cos\theta}}}^{e^{\alpha t}} \frac{2A_s A_r}{A_s^2 + A_r^2 z^2}\, dz. \tag{7}$$

This can be further simplified by substituting

$$\xi = \frac{A_r}{A_s} z \text{ and } \frac{\partial \xi}{\partial z} = \frac{A_r}{A_s} z. \tag{8}$$

$$\langle C \rangle = \left(\frac{2}{\alpha t\left(1 + \frac{1}{\cos\theta}\right)}\right) \int_{\frac{A_r}{A_s} e^{\frac{-\alpha t}{\cos\theta}}}^{\frac{A_r}{A_s} e^{\alpha t}} \frac{1}{1+\xi^2}\, d\xi \tag{9}$$

This simple integration results in the average fringe contrast of

$$\langle C \rangle = \frac{2}{\alpha t\left(1 + \frac{1}{\cos\theta}\right)} \left[\tan^{-1}\left(\frac{A_r}{A_s} e^{\alpha t}\right) - \tan^{-1}\left(\frac{A_r}{A_s} e^{\frac{-\alpha t}{\cos\theta}}\right)\right] \tag{10}$$

We now wish to determine the beam ratio that will maximize the average fringe contrast throughout the medium where the beam ratio $\propto \frac{A_r^2}{A_s^2}$. Thus we have

$$\frac{\partial\langle C\rangle}{\partial\left(\frac{A_r}{A_s}\right)} = \frac{2}{\alpha t\left(1+\frac{1}{\cos\theta}\right)}\left[\frac{e^{\alpha t}}{1+\left(\frac{A_r}{A_s}e^{\alpha t}\right)^2} - \frac{e^{\frac{-\alpha t}{\cos\theta}}}{1+\left(\frac{A_r}{A_s}e^{\frac{-\alpha t}{\cos\theta}}\right)^2}\right]. \qquad (11)$$

Setting equation 11 equal to zero gives a maximum average contrast at

$$\frac{A_r^2}{A_s^2} = \frac{e^{\alpha t\left(1+\frac{1}{\cos\theta}\right)} - 1}{e^{2\alpha t} - e^{\alpha t\left(1-\frac{1}{\cos\theta}\right)}}. \qquad (12)$$

We can partially check this equation by looking by noting that $\lim_{\alpha\to 0}\frac{A_r^2}{A_s^2} = 1$ as expected for the case of no absorption . Since $\frac{A_r^2}{A_s^2}$ is a direct indication of the measured intensity ratios, we can use this relationship to predict the optimum beam ratio for maximum average fringe contrast, thus maximum hologram efficiency.

3. FRINGE CONTRAST EXPERIMENTAL RESULTS

We recorded our edge-referenced holograms as described in the setup in Figure 2.

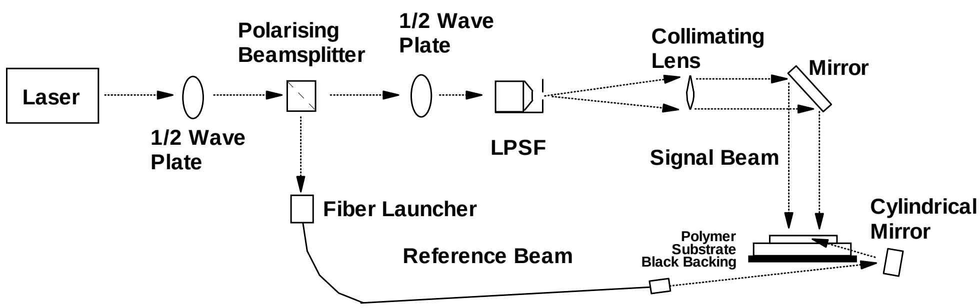

FIGURE 2. Experimental setup for the recording of an edge-referenced hologram.

In the setup depicted in Figure 2, we used the cylindrical mirror to diverge our reference as we were looking to use the virtual (non-conjugate) replay from a small light source at a short distance from the edge. We used various Du Pont photopolymers which were engineered to be red sensitive. These experimental films were a red sensitized version of the HRF 352, HRF 300x006, and HRF 700x071. Sensitization information and absorption coefficients of the various Du Pont films can be obtained from the transmission curves in Figure 3.

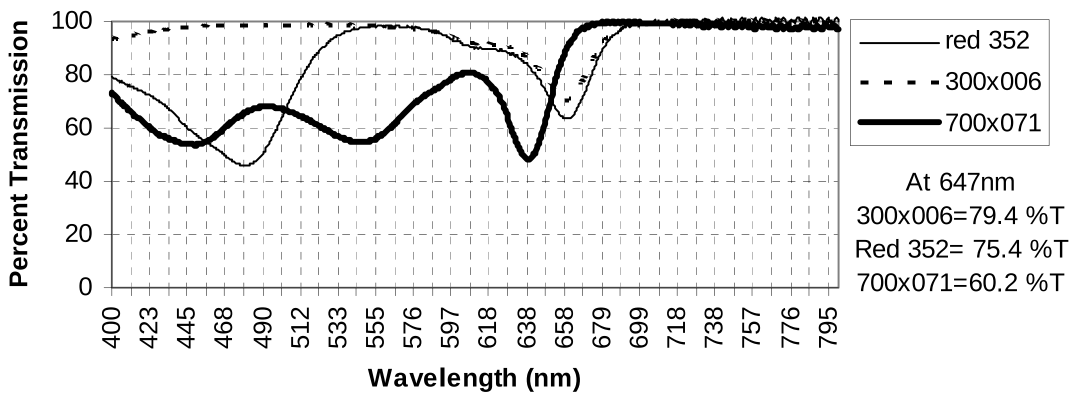

FIGURE 3. The transmission curves for 3 types of red-sensitive Du Pont Photopolymers.

With 60% transmission of HRF 700x071 at 647nm, we can deduce that it has an absorption coefficient of α=.0127μm^{-1} from the equation $\frac{I_0}{I} = e^{-2\alpha t}$ where t is the 20μm thickness of the film. We recorded the holograms at an angle of θ=78°(in the recording medium). Using equation 12, we can deduce that the optimal beam ratio is 2.6:1 (reference intensity to signal intensity), and that under those conditions, the average contrast will be 0.92.

We now need to experimentally maximize the efficiency by adjusting the beam ratio. To determine the beam ratio, we need the intensities of the signal and reference beams. The uniform signal intensity was measured directly. However, since the reference beam is Gaussian and diverges from the cylindrical mirror, we had to approximate the reference beam intensity reaching the film to determine the beam ratio. The reference intensity was estimated at a particular location at which the efficiencies were measured on a fiber based spectrophotometer. The efficiencies of the hologram were measured prior to heating due to the observation of swelling (or red-shifting) effects after heating. The efficiency values were scaled by adding 0.52 to all of the efficiencies to simulate what the efficiencies would be if the modulation simply increased during heating. This value is obtained by assuming that an average contrast of 1 would yield 100% diffraction efficiency. These scaled efficiencies are compared with the theoretical results of the fringe contrast versus beam ratio (directly from Equation 10) and are shown in Figure 4. The shape and maxima of the curve and the data demonstrate that our data correlates with the theory. The data follows the same shape as the curve and the maxima are located at the same beam ratios. This is what one would expect assuming that average fringe contrast would be directly related to efficiency.

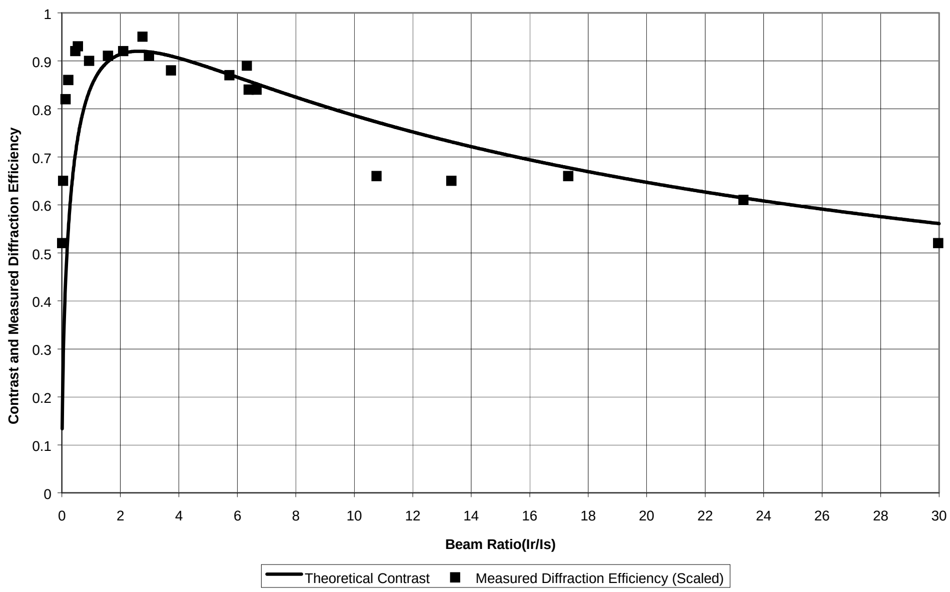

FIGURE 4. Shows that the measured diffraction efficiency is related to the theoretical prediction based on the average fringe contrast model of Equation 10. The holograms were made from the setup of Figure 2 on HRF 700x071 film and exposed to a total of 2.18mW/cm^2 at 647nm for 100 seconds followed by a UV cure.

The efficiencies were measured before heating because of the swelling effects associated with slanted fringe holograms in the Du Pont photopolymers. Since our edge-illuminated holograms have fringe angles approaching 45 degrees, the swelling effects are much more severe than in a situation of small slant angles[10]. An example of the change in Bragg wavelength is shown in Figure 5 where the holographic efficiency is indicated by a minimum in transmission.

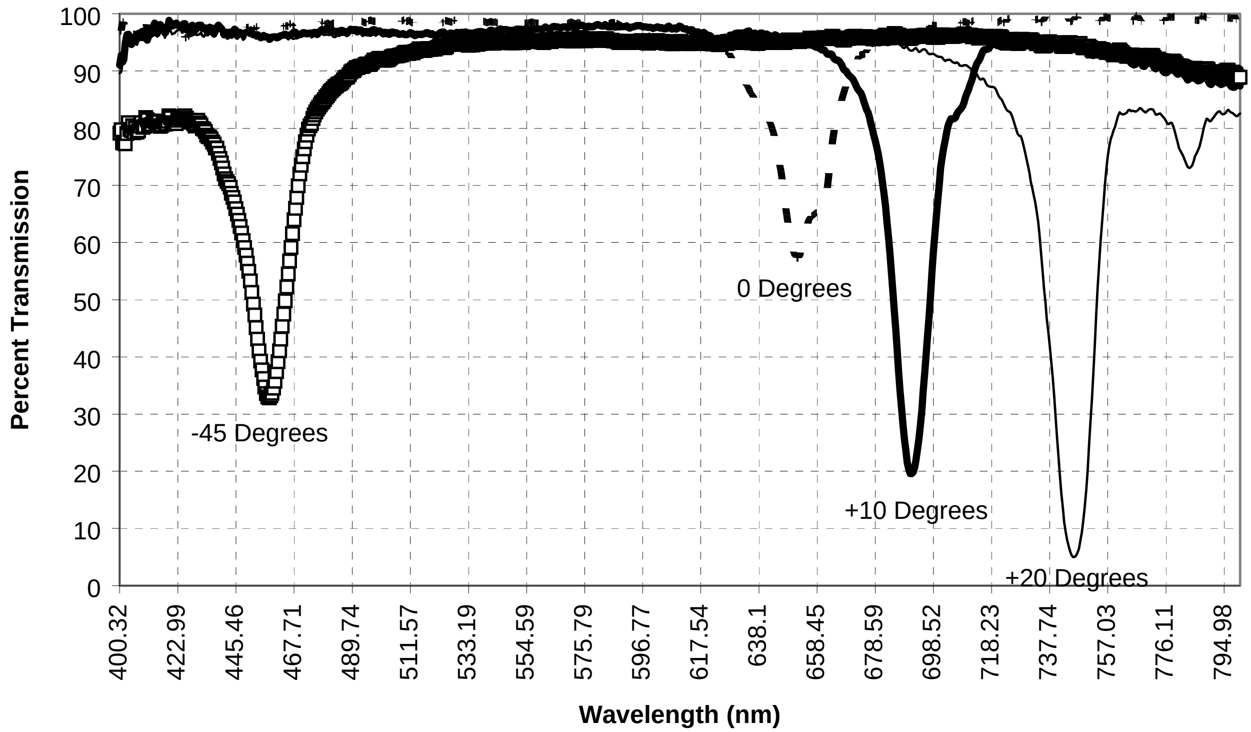

FIGURE 5. Shows the diffraction efficiency and Bragg wavelengths for an edge-illuminated hologram at various illumination angles. The hologram was heated for 2 hours at 120°C and observed in reflection at illumination angles of -45°, 0°, +10°, and +20° with respect to the surface normal (where the black light dump was during recording). A positive angle corresponds to a rotation of the illumination angle towards edge where light entered during recording.

Therefore, high efficiencies can be obtained using direct recording although one needs to pre-compensate for the swelling. Pre-compensating would require altering the recording angles and or wavelength. Inherent with the direct recording geometry is the restriction of reference angles[5]. This restriction crucially depends on the index matching of the substrate and the film. So in order to make full use of the direct recording method and use a steeper reference angle (more compact system) we have to critically monitor the index matching. We have discovered a new technique for monitoring the index matching and discuss it in the following section.

4. INDEX MATCHING AND FLUORESCENCE

The direct recording method of Figure 2 necessitates close matching of the substrate and film indices[5]. In the edge-referenced recording geometry, the fluorescence is emitted from the dye as a result of reference beam exposure. The total fluorescence of the film depends on the amount of film which is exposed. This phenomenon has enabled us to detect the conditions of precise index matching for the reference beam coupling from the substrate to the film. In the initial stage of recording we have a Total Internal

Reflection (TIR) situation and the reference beam is only penetrating into the film layer slightly according to the transverse Goos-Hänchen shift[7]. The fluorescence from this initial penetration actually decreases as one would expect due to the dye molecules breaking down or "bleaching." However, this only occurs in the evanescent layer. This penetration depth is sufficient enough to allow for the monomer to diffuse based on the generally accepted view of refractive index modulation as reported by Colburn and Haines[8], Wopschall and Pampalone[9] and Booth[6]. This diffusion increases the local refractive index in the area of the reference penetration, and thus allows further penetration because the conditions of TIR have changed. The plane of TIR would effectively be moving in the -y direction(opposite to the direction of diffusion). This Self-induced Index Matching (SIM) is indicated by a dramatic increase in fluorescence indicating precise index matching and penetration of the reference beam into the recording film. The fluorescence reaches a saturation value which indicates it has penetrated as far into the film as it can under the current conditions. This saturation occurs during the direct recording of an edge-illuminated hologram and only indicates that the since the reference has penetrated further into the film, the hologram will have characteristics closer to that of a volume hologram (as opposed to that of an evanescent or thin hologram). This phenomenon is only specific to a situation of TIR where the lower index increases to allow penetration and could apply to non-edge-illuminated geometries as well. Monitoring the fluorescence in the TIR situation may also be useful in measuring other characteristics of the photopolymer such as diffusion. An example of the fluorescence changing with time is depicted in Figure 6.

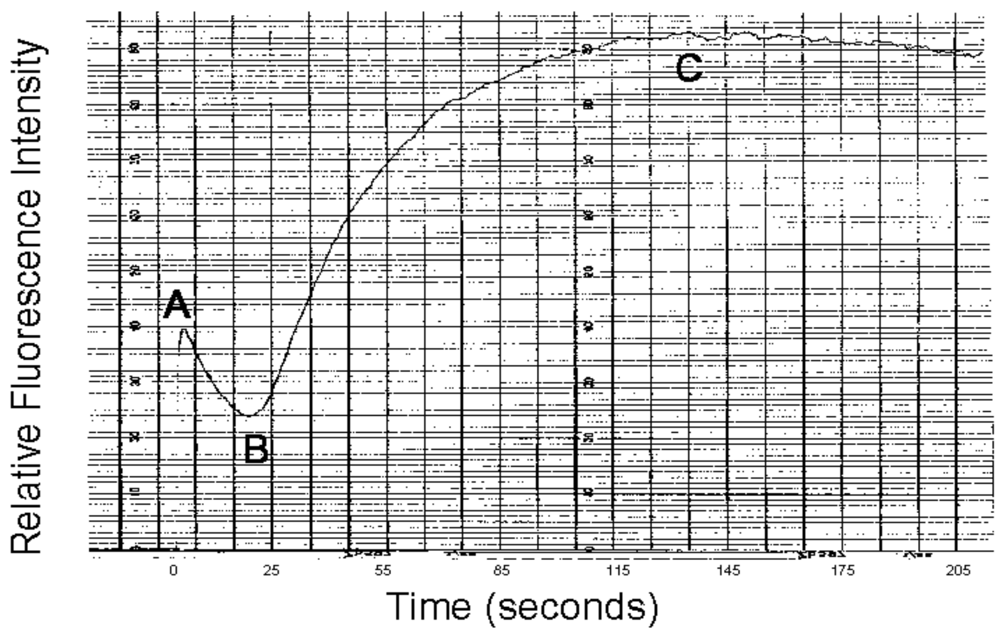

FIGURE 6. Shows the increase in fluorescence from a constant intensity reference beam for 215 seconds in the setup of Figure 2 (without the signal beam). The detector is placed behind the sample instead of the black light dump and no signal beam is used. The exposure begins at point A, and most of the diffusion begins at point B where the reference begins to penetrate through the film until point C where the fluorescence begins to saturate and gradually fall-off due to bleaching.

For this condition of precise index matching to be useful, the original indices of the substrate and photopolymer should be very closely matched. If they are not matched closely, the dynamic range of the film is spent on reaching the condition for reference penetration and none is left for the hologram.

While the fluorescence is a good indicator of index matching for the direct recording regime, there are still several complicated difficulties associated with the regime. The steep angles desired for replay force a high degree of optical quality substrates with precise indices of refraction. The fluorescence indicator can allow us to have very steep angles for recording, however, we are still restricted to a small range of angles to compensate for the swelling. In the direct recording regime, we are also restricted to the recording wavelength or forced to accept the swelled Bragg wavelength. In order to alleviate some of these restrictions we have derived an alternate recording regime which would yield the same hologram.

5. ALTERNATIVE FRINGE RECORDING GEOMETRY

One can think of a simple volume hologram formed from two collimated wavefronts as simply a set of fringes of index modulation with a specific angle and spacing. With this view, different recording regimes are possible which could create the same set of fringes, thus the same hologram.

First, we will look at the general situation which can be applied to the formation of any hologram, and then we will apply this to two different recording regimes.

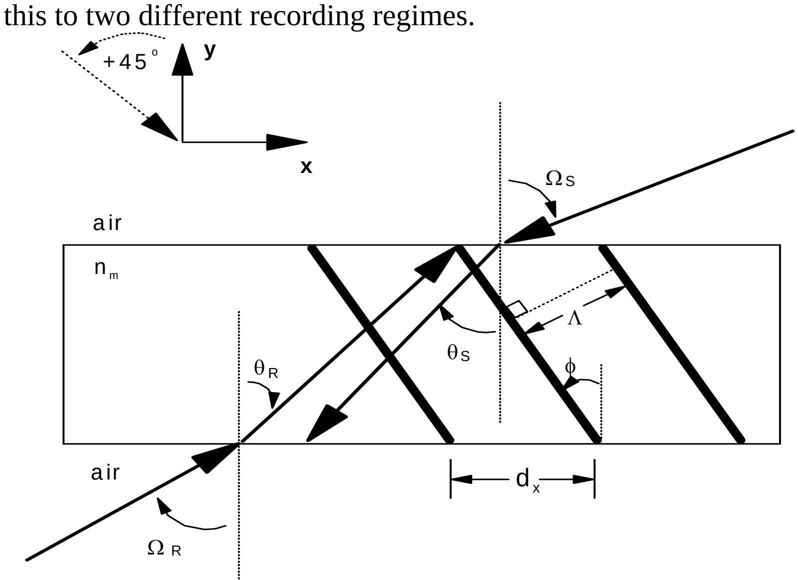

FIGURE 7. The fringes formed from a general two beam interference pattern.

Using the diagram in Figure 7, we can deduce the equation for the absolute fringe spacing(which we will call Λ) and the equation for the fringe spacing in the x-direction, d_x from Bragg's Law.

$$d_x = \frac{\lambda_m}{|\sin\theta_S - \sin\theta_R|} \tag{13}$$

Since we want to use the term for the wavelength in air, we can rewrite this using $\lambda_m = \frac{\lambda_a}{n}$ which yields,

$$d_x = \frac{\lambda_a}{n(|\sin\theta_S - \sin\theta_R|)} . \qquad (14)$$

With this we can deduce an equation for the absolute fringe spacing, $\Lambda = d_x \cos\phi$,

$$\Lambda = \frac{\lambda_a \cos\phi}{n(|\sin\theta_S - \sin\theta_R|)} \text{ where} \qquad (15)$$

$$\phi = \frac{\theta_S + \theta_R}{2} \quad \text{is the fringe angle.} \qquad (16)$$

Using the trigonometric identity

$$\sin\alpha - \sin\beta = 2\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta),$$

it can also be shown that the fringe spacing can be written in terms of the wavelength in air and the two recording angles as

$$\Lambda = \frac{\lambda_a}{2n\left|\sin\left[\frac{1}{2}(\theta_S - \theta_R)\right]\right|} \qquad (17)$$

Equations 16 and 17 give us the defining characteristics of the fringes which make the hologram. Thus if we were to make the same hologram through a different technique, it would have to have the same fringe spacing and fringe angle. We will denote the two methods by subscripts 1 and 2. Thus, $\Lambda_1 = \Lambda_2$, and $\phi_1 = \phi_2$. This leads us to the equations

$$\frac{\lambda_{a1}\cos\phi_1}{n(\sin\theta_{S1} - \sin\theta_{R1})} = \frac{\lambda_{a2}\cos\phi_2}{n(\sin\theta_{S2} - \sin\theta_{R2})} . \qquad (18)$$

Since the fringe angles must be the same, this reduces to

$$\lambda_{a2} = \left(\frac{\sin\theta_{S2} - \sin\theta_{R2}}{\sin\theta_{S1} - \sin\theta_{R1}}\right)\lambda_{a1} . \qquad (19)$$

Thus, it is possible to use two different geometries to achieve the same grating structure. We will examine the possibility of using a prism to achieve the slanted fringes while recording with a different wavelength. We will denote the geometry using the prisms with the subscript 1, and the geometry using the illumination through the edge with a subscript 2 as indicated in Figures 8 and 9.

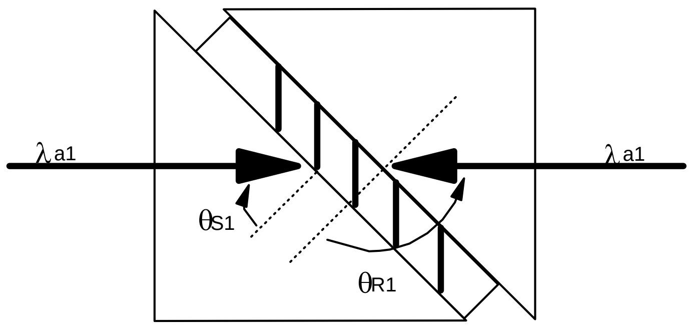

FIGURE 8. Recording geometry using two prisms

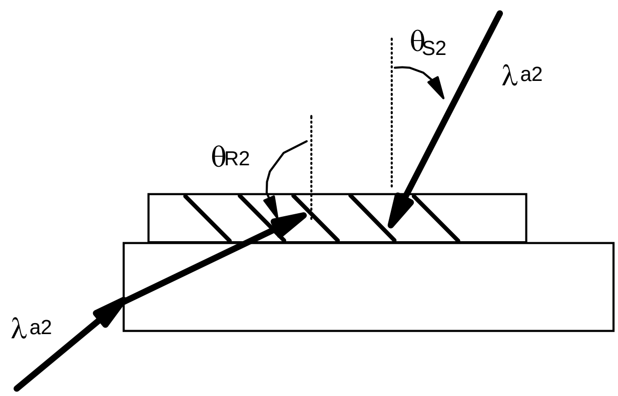

FIGURE 9. Recording geometry of the edge-referenced hologram

6. ALTERNATIVE RECORDING GEOMETRY RESULTS

For the type of edge-illuminated holograms we are developing, the geometry restricts the angles to $\theta_{S2} = 0$ and $\theta_{R2} = 98.3°$ for recording. These angles define the fringe angle to be (according to equation 16) as $\phi_2 = \frac{0 + (98.35)}{2} = 49.15°$.

Therefore, if we want to make the same hologram using the prism method, we have to meet the criterion $\phi_1 = \phi_2$. For convenience, we used a 45 degree prism and chose $\theta_{S1} = -45°$ and $\theta_{R1} = 143.3°$ which give us the same fringe angle as the edge-illuminated hologram. We can now substitute these angles into equation 19 which yields the relationship

$$\lambda_{a2} = \frac{\sin(0°) - \sin(98.3°)}{\sin(-45°) - \sin(143.3°)} \lambda_{a1} = .758 \lambda_{a1}.$$

Thus if we were to record using the prism geometry and a wavelength of $\lambda_{a2} = 647\text{nm}$ we would have a hologram equivalent to one recorded in the edge-referenced hologram depicted by the above angles and replayed at $\lambda_{a2} = (.758)(647nm) = 490nm$.

This result essentially tells us that if we record using the prism method at 647nm, we can replay the hologram at 490nm. This is of significant importance from the viewpoint of laser power in the blue wavelengths and also that beam uniformity is easier to control in the prism(alternative) method in contrast to that of the edge-referenced direct recording. Figure 10 shows the transmission spectrum of a prism recorded hologram when viewed at normal incidence(replaying as θ_s=0°). The transmission minimum for the hologram (corresponding to a reflection of light from the hologram at θ_{R2}=98.3° out of the edge of the substrate and away from the detector) is at 491nm. The hologram was made using a single beam and the two prism geometry illustrated in Figure 8 with a mirror reflecting light back at a specific angle. The single beam method was used for simplicity, however it introduced an undesirable beam ratio due to the absorption of the film. The hologram was not heated so that a Bragg wavelength shift due to swelling would not be introduced.

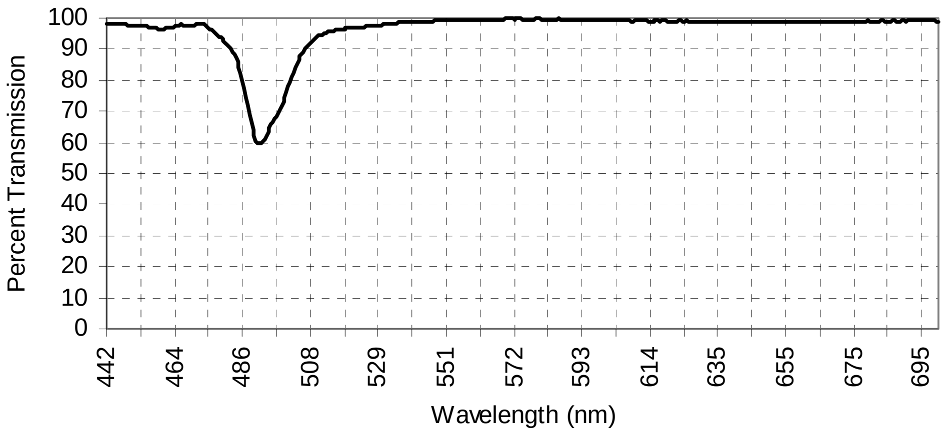

FIGURE 10. Transmission spectrum of an edge-illuminated hologram recorded using the alternative recording geometry

7. CONCLUSIONS

Direct recording of edge-illuminated holograms using an absorptive film has several inherent difficulties. In addition to the index matching, one must consider the reference attenuation. We offer an optimized fringe contrast model for producing a high efficiency volume hologram which takes into account the attenuation. The measured results follow the theoretical values for the model before the polymer is heated. When the polymer is heated we have demonstrated the extreme swelling effects for the slanted fringe holograms. While using the direct method for recording the edge-illuminated holograms, we have discovered a tool for monitoring the precise index matching between the substrate and polymer. The increased fluorescence due to reference penetration clearly illustrates the condition of exact index matching. In order to relax the conditions for the direct recording, we proposed and tested an alternative recording technique. The alternative technique offers freedom from many of the wavelength and angle restrictions of the direct method. This new technique also applies to other areas of holography where one would prefer to record at an angle or wavelength different from the intended replay.

Recording holograms with slanted fringes in Du Pont photopolymers is a complicated task. However, by taking great care in selecting the materials and processing one can help obtain good results with high efficiencies.

8. ACKNOWLEDGMENTS

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